THEORY OF ELONGATED SEPARATION ZONES

S. K. Betyaev

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Since the limit form of the flow around a body for $Re \to \infty$ is unknown within the Navier–Stokes model, turning to a simpler model of an ideal medium is the single possibility. However, another, directly opposite difficulty, an infinite set of solutions occurs during such a passage. Consequently, questions of mathematical submodelling, i.e., selection of a real flow scheme and its experimental verification, are of primary importance.

It is assumed in the theory of inviscid fluid flows that the solution is piecewise-analytical: there are slip surfaces in the domain of its existence for which vortex sheets and (or) free boundaries are selected in aerodynamics. Application of asymptotic methods to the solution of problems of discontinuous fluid and gas flows turned out to be most efficient within the framework of the theory of elongated separation zones by setting up the law of plane sections and the nonstationary analogy for the flow of a subsonic, supersonic or hypersonic flow around a body of small span τ [1]. The typical form of the internal expansion, valid in a τ-neighborhood of the elongated zones, for the potential $\varphi(x_1, y_1, z_1; \tau)$ is obtained if the time $t = z_1/w_\infty$ is introduced, where w_∞ is the unperturbed velocity directed along the z_1 axis and the transverse coordinates $x_1 = \tau x$, $y_1 = \tau y$ are stretched

$$\varphi = w_\infty^2 t + \tau^2 \Phi(x, y, t) + o(\tau^2). \tag{1}$$

Substituting the velocity components corresponding to the expansion (1) into the Navier–Stokes equation, we easily conclude that the existence of an inviscid flow mode is possible in the elongated zone if the following estimate is valid

$$Re_1 = \tau^2 Re \gg 1, \tag{2}$$

where $Re = w_\infty \ell_0/v$; v is the kinetmatic viscosity coefficient, and ℓ_0 is the characteristic zone length. For the expansion (1) to exist, the zone elongation τ should be greater in order of magnitude than the boundary layer thickness.

Known results of the theory of elongated separation and cavitation zones are obtained in flow problems [2-8]. Examined in this paper are efflux problems when there is overflow from one domain into another through the permeable plane with narrow longitudinal slots that separates them. In this case the Reynolds number characterizes the dimensionless flow rate: $Re_1 = q/v$ (q is the flow rate in a certain transverse section of the zone). The influence of viscosity is felt only in the neighborhood of the section where $q = 0$.

The jet front in efflux problems can be not only a vortex sheet but also a free boundary and a contact discontinuity. It is easy to see the validity of the nonstationary analogy even in this case: the stationary nonpenetration condition on the tangential discontinuity is converted into a nonstationary condition if the longitudinal coordinate is replaced by the time t while the condition for the pressure on the discontinuity also becomes nonstationary since the Bernoulli equation goes over into the Cauchy–Lagrange integral.

If it is sufficient to introduce one small parameter τ characterizing the elongation of the separation zone to apply the law of plane sections to flow problems, then it is necessary to take into account the presence of still another parameter ε that characterizes the pressure drop in the zone transverse section in efflux problems. Diverse possibilities for applying the method of merging asymptotic expansions appear depending on the relationship between τ and ε ($\gg$, $\approx$, $\ll$). We will call the flow for $\varepsilon = O(1)$ strong interaction between a gas with permeable boundaries while the flow with $\varepsilon = o(1)$ is a weak interaction.

Considered are problems of the flow around a narrow cutout in a screen and on the setting up of boundary conditions on a permeable wall with longitudinal slots having the period $2\tau \ll 1$.

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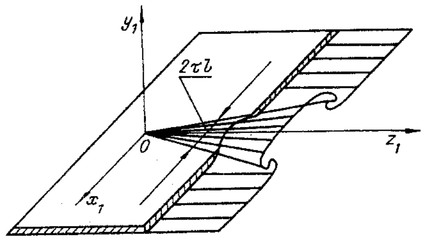

Fig. 1

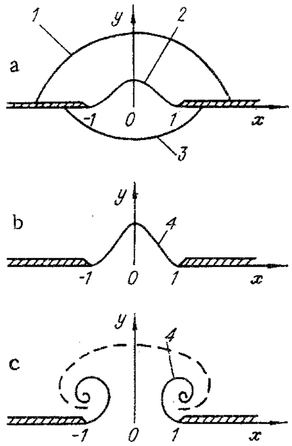

Fig. 2

The problem of gas efflux from a cutout of small span τ in a plane screen has two applications. During operation of the rear wing mechanism the diminution in the pressure drop in the neighborhood of the slots being formed results in a reduction in the efficiency of the control surfaces by spoiling the linear dependence of their aerodynamic characteristics on the angle of deviation. The influence of the slots on the reduction in the lifting capacity of the wing is usually taken into account by introducing an empirical correction factor in the value of the total characteristics [9]. The paper [10] is devoted to the separation-free flow around a rectangular cutout in a wing. The second application of the problem under consideration is the gas flow around a permeable boundary with longitudinal slots in the case of a small coefficient of permeability μ.

Let us consider the stationary flow around a longitudinal cutout symmetric with respect to a certain axis Oz_1 is a zero-thickness plane screen by an incompressible fluid. Figure 1 shows the triangular cutout. The origin is at the apex of the cutout, the Oy_1 axis is directed along the normal to the screen and the Ox_1 axis along the screen. The shape of the cutout edges has the form $x_1 = \pm\tau\ell(z_1)$. Quantities associated with the fluid particles passing above and below the screen are denoted, respectively, by the superscripts + and −. These particles are separated by the plane of the screen and an interfacial surface resting on the edge of the cutout.

The outer expansion describes the flow around the screen (wing) with multipole singularities on the segment $0 \le z_1 \le \ell_0$, $r_1{}^2 = x_1{}^2 + y_1{}^2 = 0$. The first term of the outer expansion, i.e., the solution of the problem in the absence of the cutout ($\tau = 0$) will be considered known

$$\lim_{r_1\to 0} \varphi^{\pm}(x_1, y_1, z_1; \tau) = \varphi_0^{\pm}(z_1). \tag{3}$$

The influence of the cutout on the flow as a whole is localized if the pressure drop on the screen surface is small

$$p^+ - p^- = p_0^+ - p_0^- - \frac{1}{2}\rho\left[\left(\frac{d\varphi_0^+}{dz_1}\right)^2 - \left(\frac{d\varphi_0^-}{dz_1}\right)^2\right] = \varepsilon\rho b_0(z_1). \tag{4}$$

Here ρ is the fluid density, and $p^{\pm}$ is the stagnation pressure.

The physical meaning of the parameter $\varepsilon \ll 1$ is determined by the specific conditions of the problem. For instance, within the framework of linear wing theory the parameter ε will be the angle of attack. The relation between the parameters ε and τ is set up during merging the exterior and interior expansions.

Condition (4) allows of different interpretations, out of which we consider the case when the difference between the total pressures is small ($p_0{}^+ - p_0{}^- = O(\varepsilon)$) and the inner limit of the outer expansion (3) is

$$\lim_{r_1\to 0} \varphi^{\pm}(x_1, y_1, z_1; \tau) = w_\infty z_1 + \varepsilon\Phi^{\pm}(z_1) + o(\varepsilon). \tag{5}$$

Let us investigate the flow of a single-phase fluid when the tangential discontinuity is a vortex sheet that originates in conformity with the Chaplygin–Zhukovskii condition

Fig. 3

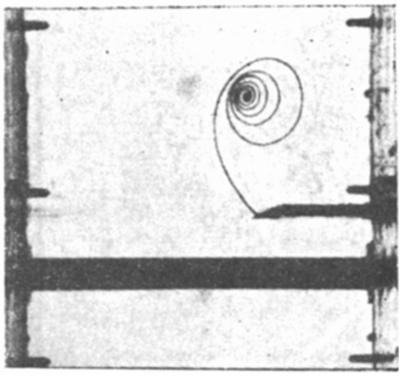

Fig. 4

about the finiteness of the velocity at the sharp edges of the cutout and consists of particles passing through the edge. Firstly it is clear that the flow rate q(t) depends on the pressure drop on the screen determined by the quantity $\Phi^- - I^+$.

The inner expansion differs from (1) by the intermediate term

$$\varphi(x_1, y_1, z_1; \tau) = w_\infty^2 t + \varepsilon\Phi_0(t) + \tau^2\Phi(x, y, t) + o(\tau^2).$$

The outer limit is

$$\lim_{y\to\pm\infty} \Phi(x, y, t) = \pm\frac{q}{\pi}\ln\frac{r}{l_0} \qquad (6)$$

($r_1 = \tau r$, ℓ_0 is the cutout length). Comparing the limits (5) and (6) we find $\varepsilon = k\tau^2 \ln\tau$, $2q = k\pi(\Phi^- - \Phi^+)$, $2k\Phi_0 = \Phi^+ - \Phi^-$.

The theory of elongated and separation zones is valid if the similarity parameter is $k = O(1)$, for $k \ll 1$ the linear efflux theory [11] is valid, while for $k \gg 1$ the problem remains three-dimensional. Such a formulation differs substantially from that studied in [12] where out-of-order terms are contained.

The next term of the outer expansion determined by the limit $\pm\tau^2(q/\pi)\ln(r_1/\ell_0)$ corresponds to flow from linear sources ($y_1 > 0$) and sinks ($y_1 < 0$) located on a segment of the z_1 axis.

The inner expansion in the principal approximation describes the plane nonstationary fluid efflux from a slot of variable width $2\ell(t)$. The method of solving this problem and certain solutions are presented in [11], where the reverse vortex sheet and the cumulative effect of a self-similar jet are discussed. The simplest flow schemes, including the efflux of a compressible gas (a) are shown in Fig. 2 (1 is the shockwave; 2 is the contact discontinuity; 3 is the rarefaction wave; and 4 is the vortex sheet). If the cutout has a sharp apex, i.e., $\ell(0) = 0$, then the flow scheme is possible when all the fluid particles forming the jet front are vortices (b). If the cutout has a leading edge, i.e., $\ell(0) \neq 0$, then the particles passing through it will remain nonvorticized in conformity with the Lagrange theorem (dashed line in Fig. 2c), the jet front has a mushroom shape with free ends of the vortex sheet curled up into two spirals.

Efflux problems are characterized by a large diversity of schemes that depends substantially on the initial data and which are not always successfully set up a priori before performing the numerical computations or tests. Presented in Fig. 3 is a photograph on the initial stage in the buildup of a stationary efflux from a constant width slot, experimental verification of the scheme displayed in Fig. 2c. The spiral structure of the flow is disclosed upon feeding dye to the slot edge in a vertical water tunnel (Fig. 4). There is no second edge here, and it can be considered that the flow in a half-slot is investigated here. The efflux pattern is similar to a Prandtl acceleration vortex.

The plane nonstationary problem was solved numerically for different kinds of function ℓ and fluid flow rate through the slot q. Shown in Fig. 5 is the dependence of the

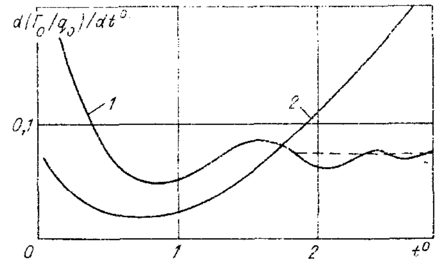

Fig. 5

circulation growth rate of one spiral chuck of the vortex sheet Γ_0 on the dimensionless time $t^0 = tq_0\ell^{-2}$ for a symmetric efflux from a slot of constant width (ℓ = 2) for different values of the function $q(t^0)$: 1) constant flow rate, $q = 2q_0$; 2) linearly growing flow rate $q = 3q_0t^0$.

In the first case the flow near the slot emerges into the stationary mode (dashed line in Fig. 5) in conformity with the Helmholtz scheme. The numerical solution of the nonsymmetric problem in the presence of downwash is obtained in [13].

During the stationary interaction of two fluids one sets the other into motion just by using tangential stresses. A thin mixing layer forms on the boundary of slightly viscous medium that ejects a small quantity of the gas in the rest phase. Consequently, it is natural to assume that the contact surface of inviscid media is a free boundary, on which the pressure is constant. Such a flow scheme has important application in a study of gas flow interaction with permeable walls of a wind tunnel working section.

Needed for the active control of the flow in a tube and the computation of its induction is knowledge of the boundary conditions resulting from the initial equations (the first principles of physics), and taking account of the mechanism of gas interaction with a permeable wall. The empirical condition $\varphi_{z_1} = \alpha\varphi_{y_1z_1}$ ordinarily utilized [14] contains out of order terms (the constant $\alpha \ll 1$) and does not take account of the separation interaction between the gas flow and the wall. The large series of computations executed are underrated because of the utilization of empirical boundary conditions.

If the number of slots per unit width of the permeable wall section is known, then the boundary conditions on the wall cannot possibly be obtained, and the problem of the flow around a profile should be solved jointly with the problem of determining the free surface shape and intensity that rests on the sharp edges of the slot. Formulation of the boundary conditions is possible if the slot half-period τ is small. In this case the periodic flow in the τ-neighborhood of the permeable wall is equalized during emergence from this neighborhood into the working section and its certain averaged characteristics are the missing boundary condition for the external problem about the flow around a body in a wind tunnel.

Let us consider permeable walls with periodic longitudinal slots of width $2\tau \ll 1$. There is no overflow through the permeable walls in the absence of a profile in the tube, i.e., the pressure in the plenum chamber equals the static pressure in the unperturbed flow.* Then the maximal profile thickness can be selected as the parameter ε that characterizes the pressure drop in the near-wall layer. For $\varepsilon = 0$ there is no pressure drop. An inviscid efflux mode [15] is realized in the neighborhood of permeable walls since the estimate $\varepsilon\tau \mathrm{Re} \gg 1$, replacing condition (2), holds.

For weak interaction ordinary linear expansions of the subsonic or supersonic theory of a profile $\varphi = w_\infty z_1 + \varepsilon\varphi_0(y_1, z_1) + \varepsilon^2\varphi_1(y_1, z_1) + o(\varepsilon^2)$ with nonpenetration conditions on the body, on the permeable wall sections of the tunnel, and with the condition of low homogeneity as $z_1 \to -\infty$ are varied in the outer expansion. The desired boundary condition on the permeable section $y_1 = h$ ($a_1 \le z_1 \le a_2$) of the walls should be determined from the condition for merging with the inner expansion which is valid in a small neighborhood of the wall, where the outer expansion is not suitable because of the three-dimensional nature of the flow. The formalism of the expansion is different from that considered above in that the flow in the neighborhood of the permeable boundary is periodic in the transverse direction (along the x_1 axis). The nonpenetration condition is satisfied on the flow domain

*Taking account of the variable pressure in the plenum chamber, i.e., on the free surface, introduces no difficulties in principle.

boundaries, the infinite strips ($-1 \le x \le 1$, $-\infty < y < \infty$) with the two slits ($|x| > \mu$, $y = 0$), by virtue of symmetry. In place of (1) we have

$$\varphi(x_1, y_1, z_1; \tau) = w_\infty z_1 + \varepsilon^2\Phi_0(z_1) + \varepsilon\tau\Phi(x, y, z_1) + o(\varepsilon\tau), \tag{7}$$

where $y_1 = h + \tau y$; $h = O(1)$.

Substituting (7) into the Euler equation, we find that in conformity with the theory of elongated separation zones the potential satisfies the Laplace equation $\Phi_{xx} + \Phi_{yy} = 0$, i.e., compressibility is not essential.

Since the longitudinal velocity perturbation in the outer expansion is larger in order of magnitude than the longitudinal velocity perturbation in the inner expansion, we obtain Nikol'skii's boundary condition [16] on the permeable wall from their merger

$$\frac{\partial\varphi_0(h, z_1)}{\partial z_1} = 0. \tag{8}$$

The external problem with condition (8) becomes linear, its solution is obtained in closed form [17] and determines the velocity $v_0(z_1) = \partial\varphi_0(h, z_1)/\partial y_1$, i.e., the gas flow rate through the slot needed as the boundary condition for $y = -\infty$ to solve the inner problem. However, the difference of the flow in the working section of the wind tunnel with longitudinal slots from the flow in a free jet (in an Eiffel chamber) on whose boundary condition (8) is also satisfied because of the constancy of the pressure in not apparent. Also not apparent is the dependence of the solution on the slot geometry (the coefficient of permeability μ, etc.). Consequently, to determine the tunnel induction it is necessary to construct the next approximation including the solution of the internal problem.

If $\varepsilon \ll \tau$, then the linear theory of free boundary evolution is valid. If $\varepsilon = O(\tau)$, then the problem remains nonlinear and $\Phi_0 = 0$ can be assumed. The outer limit

$$\lim_{y\to-\infty} \Phi(x, y, z_1) = yv_0(z_1) + b(z_1). \tag{9}$$

is determined as a result of solving the internal problem.

The inner expansion (the function b) determines the pressure drop needed for a given fluid mass flow rate to penetrate through a periodic lattice of slots, the longitudinal velocity of the external flow is adjusted under the pressure on the wall determined in such a manner. From the merger with the limit (9) we have a boundary condition for the second approximation of linear profile theory

$$\frac{\partial\varphi_1(h, z_1)}{\partial z_1} = b'(z_1). \tag{10}$$

Therefore, the problem of the flow around a profile in a channel with longitudinal slots reduces to two plane problems: a linear stationary problem for the flow around a profile by a compressible gas, and a nonlinear nonstationary problem for incompressible fluid flow around a lattice of slots. A typical nonstationary efflux diagram is shown in Fig. 6, where the jet front is the free boundary.

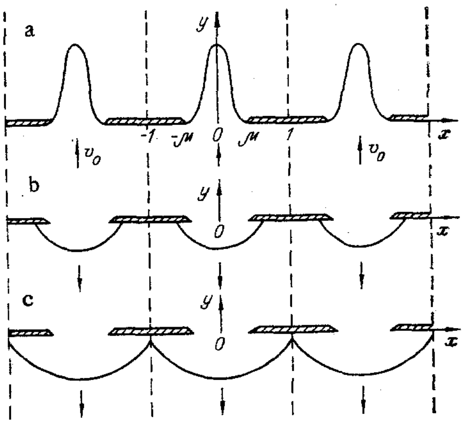

Fig. 6

Let us still consider two cases when the separation zone is flattened and the nonstationary analogy is inapplicable. If $\tau \ll \varepsilon \ll 1$, then in contrast to the diagram displayed in Fig. 6a, the jet front in the scale of the inner expansion leaves for infinity since it has the length $O(\varepsilon)$ exceeding the characteristic thickness of the near-wall layer $O(\tau)$. The flow is stationary, and the longitudinal coordinate, rather than the time, plays the part of the parameter. In a first approximation condition (8) is again valid. Considering the coefficient of fluid jet compression σ a given function of μ and utilizing the Bernoulli equation to determine the pressure in the jet for $y = \infty$, we obtain in a second approximation

$$w_\infty \frac{\partial \varphi_1(h, z_1)}{\partial z_1} = -\frac{v_0^2}{2\sigma^2\mu^2} \quad (\partial\varphi_1(h, z_1)/\partial z_1 = \Phi_0'). \tag{11}$$

Condition (11) is linear since its right side is considered a given function of z_1 in the determination of the second approximation (the potential φ_1). Therefore, in the weak interaction case, the theory of small perturbations results in a natural manner in linear boundary conditions, and attempts to deduce nonlinear boundary conditions (see [16], Ch. V, say) should be acknowledged inconsistent. Condition (11) is valid only on the section of the flow out of the working section into the plenum chamber, whose end is determined from $\partial\varphi/\partial y_1 = 0$ for $y_1 = h$. The flow diagram changes substantially at the inflow section (Figs. 6b and c) while the dependence on x vanishes in the intermediate layer $y_1 = h + O(\varepsilon)$. Therefore the flow can be considered planar. The boundary condition for the exterior problem is constancy of the pressure and the stream function on the unknown free boundary located in the working section.

In the case of a strong interaction, which is of especial interest for practice in connection with a marked tendency to increase the Re in wind tunnels, τ remains the single small parameter. The external problem is nonlinear. The inner expansion (7) is rewritten in the form

$$\varphi(x_1, y_1, z_1; \tau) = \Phi_0(z_1) + \tau\Phi(x, y, z_1) + o(\tau). \tag{12}$$

All the velocity components in (12) are identical in order of magnitude. The longitudinal velocity component Φ_0' is independent of the transverse coordinates. Substituting (12) into the three-dimensional equations of motion of a perfect gas with adiabatic index γ we find the equation of the plane sections

$$\begin{gathered}(\Phi_x^2 - a^2)\Phi_{xx} + (\Phi_y^2 - a^2)\Phi_{yy} + 2\Phi_x\Phi_y\Phi_{xy} = 0,\\ a^2 = a_0^2 - \frac{\gamma-1}{2}(\Phi_0'^2 + \Phi_x^2 + \Phi_y^2),\end{gathered} \tag{13}$$

which describes, with the appropriate boundary conditions, the stationary symmetric jet gas flow around a lattice of slots in the plane z_1 = const. The sound speed at the flow stagnation point depends on the longitudinal coordinate z_1 which also plays the part of a parameter here. The boundary condition is nonlinear, is obtained as a result of merging the inner expansion (12) with the exterior describing the flow in the y_1, z_1 plane. To do this it is sufficient to know the jet compression coefficient σ and to use the isoentropicity condition and the Bernoulli equation in order to determine the pressure p_1 in the jet as $y \to \infty$, which equals the given pressure in the plenum chamber

$$\left[\frac{\partial\varphi(h, z_1)}{\partial z_1}\right]^2 + \frac{1}{\sigma^2\mu^2}\left[\frac{p(h, z_1)}{p_1}\right]^{1/\gamma}\left[\frac{\partial\varphi(h, z_1)}{\partial y}\right]^2 = \frac{2a_0^2}{\gamma-1}\left[1 - \left(\frac{p_1}{p_0}\right)^{\frac{\gamma-1}{\gamma}}\right] \tag{14}$$

(p_0 is the stagnation pressure). Condition (14) is valid only on the outflow section. In the general case the outer flow will be three-dimensional, periodic along the x_1 axis, in the inflow section.

Therefore, depending on the relationship between the parameters ε and τ the application of asymptotic methods to the problem of gas flow in a wind tunnel with longitudinally slotted walls is possible in four cases. For a strong interaction it is necessary to take account of compressibility in the interior domain, the boundary condition (14) is valid, while for a weak interaction the gas compressibility is not essential and the Nikol'skii's condition is valid in a first approximation. To obtain the second approximation needed for $\tau \ll \varepsilon \ll 1$ it is necessary to use condition (11), to solve the linear problem on evolution of a free boundary in the near-wall layer for $\varepsilon = o(\tau)$, and to solve the nonlinear problem on free boundary evolution for $\varepsilon = O(\tau)$.*

*The numerical solution of such a problem in combination with the problem of the flow around a profile is an extremely difficult problem at the present time.

In the last two cases the free surface does not emerge from the elongated zone, the boundary condition (10) is substantially an integral taking account of the effect of "memory" on the slot geometry and on the separation nature of their flow up to the section z_1 = const, which is a consequence of the fact that the Cauchy problem [18] is valid for the inner expansion. This, in principle, is the difference between the mechanism of gas interaction with longitudinally slotted boundaries and the mechanism of gas interaction with transversely slotted or perforations.

Each of the four expansions proposed and their corresponding boundary conditions has its advantages and disadvantages. Each of them turns out to be most effective, can only be clarified because of extensive experimental and calculational exploitation.

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